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Electric Power Consortia: Toward Decision Support Based on Market Optimization

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Abstract

Proposed in this paper is an extensible decision support system framework to facilitate Commercial and Industrial entities forming a consortium to collaborate on their electric power supply and demand in order to streamline their consumption and reduce their costs. The collaborative framework includes the structure of market setting, participants' bids, and a market resolution which produces a schedule of how power components are controlled as well as the resulting payment by market participants. We also define four properties that the market resolution must satisfy, namely, *feasibility*, *Pareto-optimality*, *Nash equilibrium*, and *equal collaboration profitability*. Furthermore, we develop a market resolution algorithm, based on a formal optimization model and prove that it satisfies the desirable market properties.

1 Introduction

There has been an ongoing trend of moving toward less reliance on conventional hydrocarbon energy resources and more adoption of cleaner alternative energy due to increased fuel costs or to be more sustainable. This trend created a plethora of alternatives that promise to cut carbon emissions and pollutants. Commercial and Industrial (C&I) organization have a variety of power enabled services. Furthermore, they add a variety of energy and power resources including Photovoltaics, wind, storage, local back-up generation, and commercial contracts on supply of power and load curtailment.

In this context two complex questions arise: (1) how to optimally operate available resources over time, and (2) how multiple C&I organizations can collaborate on sharing resources to minimize their costs. This paper focuses on decision support for C&I organizations to address these two problems.

To better understand interaction and collaboration between different units, consider an illustrative scenario depicted in Figure 1. In this scenario multiple units (e.g., C&I organizations) have a diverse set of resources that supply power and provide multiple services that consume power at any operation time interval.

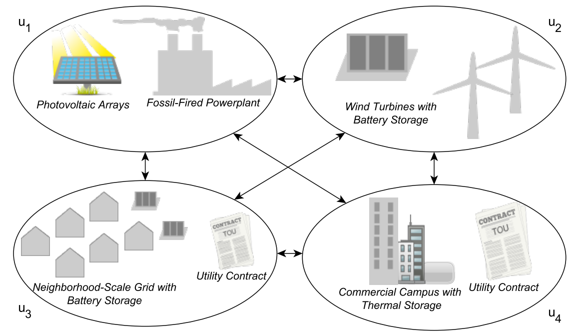

Figure 1: Power Loads & Resources Collaboration Example

While a contract with electrical power company is common, an organization can have other energy resources (e.g., photovoltaic power systems, storage batteries, backup engine generators, etc.) at its disposal. With so many alternatives, finding the optimal operation of such resources while taking into consideration the possibility of collaborating with others becomes an increasingly complex problem.

In order to be able to model such a scenario, we must define how electrical power components (i.e., electrical power resources, or electrical power consuming services) are modeled. Furthermore, we need to describe how they behave under various conditions. In general, power components of the electrical power infrastructure can either produce power, consume power, store power, or remain idle at any given time interval over the time horizon under consideration. While power components' internal workings can be unique, we try to find common characteristics that these components share in order to generalize their modeling.

Since the introduction of the deregulated electric power market, many innovative solutions have been proposed to increase competitiveness and reduce costs. As discussed in more detail in Section 2 (Related Work), a range of demand response (DR) techniques have been proposed [1]. DR techniques try to adapt consumption based on changes in energy supply. Also, power auction markets have been used to increase supply side competitiveness and price variability based on changes in demand [2]. Furthermore, there have been efforts to model dynamic real-time allocation of resources such as wireless spectrum between participating wireless carriers with promising results [3]. However, these concepts did not address (1) how participants of a consortium can collaborate to share power resources and services (2) how gains from such collaboration can be fairly distributed among the participants.

In previous work [4, 5], we explored the idea of how units of an organization can optimally share their peak-demand bounds to achieve an overall better operational utility while fairly compensating participating units. We also optimally planned the selection of peak-demand of collaborating entities based on the projected demand over a time-horizon with the condition that units can share their peak-demand bounds. Both approaches resulted in a better overall optimal level than if each player acted separately. However, this approach does not consider a range of power resources and services (e.g., photovoltaics, battery storage, backup generator, etc.), and is not extensible. Bridging this gap is the focus of this paper.

Making decisions in an environment where the the benefit received from the operation of power consuming services and the costs of power generation and acquisition coupled with the possibility of collaboration in real-time and for future planning with other C&I units presents a complex problem. The purpose of this paper is to introduce a decision support framework (see Figure 2) where C&I units that demand power, supply power, or both can collaborate to optimize the operation, generation, and acquisition of electrical power components so that they achieve a better financial and operational level.

In this paper, we propose an extensible Decision Support System framework for market-based collaboration of power resources and services. In doing so, we create an extensible model where resources & services can be added or removed by minimally describing their attributes. More specifically, the contributions of this paper are as follows:

First, we propose and formally define a collaborative market framework. The basic idea of this market is to create a consortium of organizational units where each unit has the freedom to make decisions related to power consumption, generation, and storage. Members of this consortium have some services that they need to run (e.g., lighting, HVAC, water heating, etc.) and also have some power resources (e.g., utility contract, photovoltaics, backup power generator, etc.). The members also have some expectation of the intrinsic value of running services at different levels of operation over a time horizon represented as a bid that each member of this consortium submits to market. The market resolution produces a power resource allocation for each unit, and the payment that each member has to pay or receive. We also define four desirable properties that our market must satisfy, namely, *feasibility*, *Pareto-optimality*, *Nash equilibrium*, and *equal collaboration profitability*.

Second, in order to support market resolution, we develop and implement a formal optimization model to decide on the operation of resources to be used, and the services that are run while maintaining feasibility, i.e, the power consumed by all members of the consortium does not exceed the total power supply.

Third, we develop a market resolution mechanism based on the optimization models that guarantees the satisfaction of the defined properties of market, namely, *Pareto optimality, Nash equilibrium*, as well as the property of *equal collaboration profitability*, defined formally in this paper.

This paper is organized as follows: In the second section, we present a brief survey of related work. In the third section we present a small example of the collaboration problem. In section four, we describe our collaborative market framework where we formally define the power market setting, market bids, market resolution and desirable properties that our market must satisfy. In section five, we describe the market resolution algorithm and how the extra benefit that resulted from the collaboration is fairly distributed among the participants. Finally, we briefly discuss our conclusions and give some future work directions.

2 Related Work

Power markets have been the focus of a great deal of research. There have been numerous efforts to deal with reducing power consumption costs either through improving technological efficiency or through market supply and demand mechanisms. An extensive work was directed to dealing with the reduction of power consumption from the demand side by changes in price in an area that is termed Demand Response (DR) [1]. This approach has been used broadly with large power consumers to cut or curtail demand when power generation and transmission networks are about to be overloaded. The participants are motivated by promising financial incentives if they comply. Demand response entails changing the consumers normal consumption patterns in response to changes in price, or to qualify for a certain incentive payment. Such DR programs are categorized into two broad groups: price-based, and

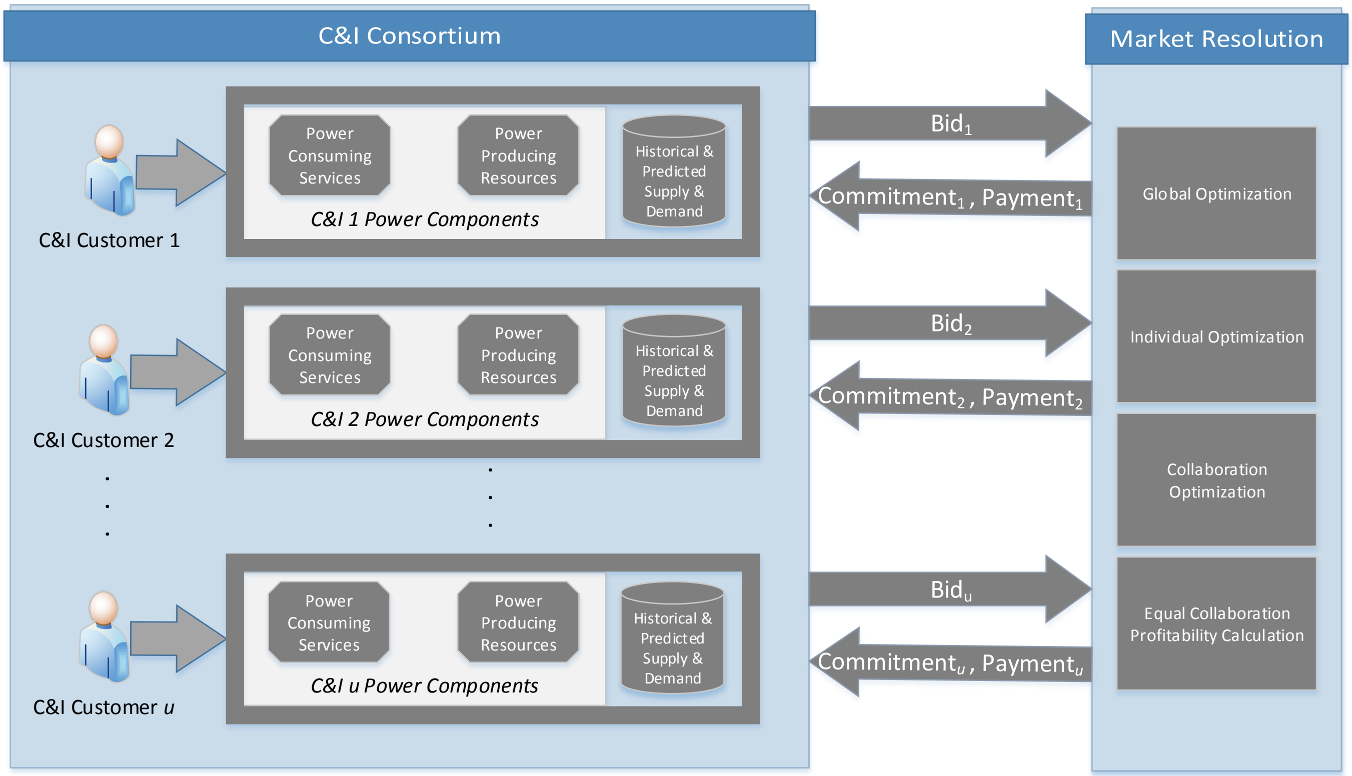

Figure 2: Electric Power Collaboration Decision Support Framework

incentive-based. Price-based methods include the use of time-of-use (TOU) rates, real-time pricing (RTP), and critical peak pricing (CPP). Whereas, incentive-based methods use techniques such as: direct load control (DLC), interruptible/curtailable (I/C) service, demand bidding/buyback (DB), emergency demand response programs (EDRP), capacity market programs (CMP), and ancillary services market programs (ASMP). These methods are summarized in [6].

There has also been significant work on deregulated electricity markets and their competitive characteristics [2]. Most of the work is has been directed to different parts of market design, mainly, the relationship between power generation companies and wholesale companies. Different approaches to the use of auctions in electric markets has been investigated, e.g., in [7], [8], [9],[10], [11], and [12]. They discuss methods in how such markets should be designed to account for buyers and sellers of electric power. Although there has been some work that tries to address large consumers power procurement optimization by evaluating different procurement options [13], they fall short of addressing modeling internal power components and collaboration optimization. There has been also some effort to control peak demand and reduce overall consumption by using physical improvements [14] and load scheduling [15] yet the idea of designing a decision support framework for the power components modeling and collaboration has not been addressed.

Furthermore, such concept of dynamic allocation have been employed in other fields like computational systems resource distribution and wireless spectrum allocation (i.e., [16, 17, 18, 3, 19, 20]). Solutions of microeconomic equilibrium have been implemented with promising results. However, the notion of using such methods to share load consumption and power resources to allocate power among multiple participants such as units has not been explored. Moreover, studied electricity markets are akin to commodity markets with special characteristics whereas in our case, we need to consider power components with right to use, which is more akin to options market.

There have been several attempts to create a real-time pricing market where participant place bids at every considered time slot. The ideas propose models for collaboration between customers of a power company where participants place bids that correspond to benefits gained from running household appliances. It also considers a mix auxiliary power sources such as batteries and plug-in hybrid electric vehicles (PHEV). With simulated loads, the results indicated an overall stabilization of power consumption curve over the considered time span compared to flat-rate which resulted in the reductions of peak-demand consumption. While such solution is promising, it does not consider the dynamics and the cascading effects of power components planning for units of an organization or a consortium of organizations where certain resolution and and payment exchange must be determined at each time interval while optimizing for the entire time horizon. It also doesn't propose a fair mechanism to sharing the extra benefit of collaboration versus working alone [21].

In previous work [4] we investigated the optimal distribution of an already allocated peak demand among participating organization's units. Under such model the peak-demand budgets of participating units were pooled together to increase the total welfare of an organization. A fair compensation and payment method was proposed to distribute gains from collaboration. We have also proposed to account for short term variability of renewable resources and employ this variability to make decision for peak-demand distribution. Short term knowledge of efficiency that can be gained from the immediate variability of renewable was employed for the exchange of peak demand power bounds between participating entities. We also proposed a model of how to determine and optimal peak demand limits given the opportunity of collaboration with other participants later and how gains from reducing the contracted peak demand bounds are fairly distributed among participants [5]. In this work however, we try to generalize and expand our model to account for any type of power component (e.g., consumption, generation, or storage) given the component's attributes.

3 Problem Example

To make the problem more concrete consider two an example depicted in Figure 3 with Unit 1 and Unit 2. Unit 1 runs two power consuming services, water heating, and HVAC. These two services have value to their respective unit which we call intrinsic value (measured in dollars amount). Each power consuming service needs power to operate measured in kW. By intrinsic value we mean the amount in dollars that a unit is willing to accept in lieu of shutting that service off. Unit 1 also has two types of power resources. The first is a utility contract with a power company. This is not a power resource in itself but a right to use power on agreed upon terms. Utility contract may state the rate per kWh in dollars and a maximum peak demand consumption level before incurring a penalty rate. The other type of resource is a backup power generator. Unit 2, on the other hand, has two other power consuming services: lighting, and Plug-in Electric Vehicle (PEV) charging . Unit 2 resources are Photovoltaic unit and battery power storage unit. The battery unit has controls that can be instructed at any given time interval to either store power (charge), provide power (discharge), or stay idle.

Using a power resource typically incurs certain cost which can be either variable, fixed or both (e.g., acquisition cost, fuel cost, maintenance cost, etc.). If these resources are dispatched to third parties, they can generate revenue. These resources usually have certain status indicators (e.g., charge level, efficiency, etc.) depending on the type of resource. Resources also have constraints that determine the feasible operation parameters depending on multiple factors including the status of the power resource. Most power consuming and producing components allow for control that affect their operation.

In a typical environment, units operate independently to satisfy their power loads. Now let's consider a scenario where unit 1 has a previously unanticipated surge in demand. Unit 1 now has many alternatives to consider. It can exceed its peak demand and incur penalty which could affect the entire contractual period. It can also arbitrarily curtail demand without giving much thought to the lost intrinsic value of the service being turned off. The unit can also dispatch the battery to satisfy excess demand without considering the diminished ability of the battery to satisfy possible future demand. Now add to that the ability for multiple units to collaborate. That means if units 2 would agree to a certain compensation, unit 1 could tap into unit 2's power that is coming from the either power company or use unit 2's backup engine generator. Choosing an alternative that maximizes the unit's value becomes increasingly complex without a collaborative power resource and load sharing market framework.

The formal market framework described in the next Section (4) is designed to address the problem of (1) how exactly each combined resource and service operate (2) How to correctly compensate some units for enabling the usage of their resources.

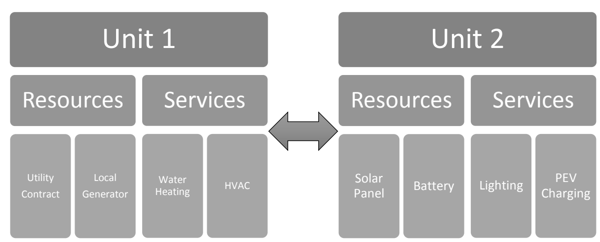

Figure 3: Example Problem Units' Power Components

4 Collaborative Market Framework

In this section we define the power market setting, market bids, and the market resolution and its desired properties. We begin by describing the power market setting.

4.1 Power Market Setting

To facilitate market mechanisms, we assume that the market consist of a set of components $C = \{1, \cdots, n\}$. Power components can be in the form power producing resources such as diesel generators, solar panels, etc. They can also be in the form of power consuming services such as lighting, air conditioning, water heating, etc. The time horizon is a set $T = \{1, \cdots, N\}$, i.e., we assume that time is divided into discrete time intervals which determine the market execution frequency. For example, a day of operation can be divided into 24 hours, i.e., $N = 24$.

A control vector $a_{i,t}$, $1 \leq i \leq n$, $1 \leq t \leq N$ represents the control actions that component i takes at time interval t. Let $\mathrm{dom}(i)$ indicate the domain of control values for component i. A vector of controls $\bar{a}_i = (a_{i,1}, \cdots, a_{i,N})$, $1 \leq i \leq n$, represents the control actions that component i takes over the time horizon N. The control actions for all components over the time horizon N is represented as matrix

$$A = \begin{pmatrix} \bar{a}_1 \\ \vdots \\ \bar{a}_n \end{pmatrix} = \begin{pmatrix} a_{1,1} & \cdots & a_{1,N} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,N} \end{pmatrix}$$

We assume that the market consists of a set of units $U = \{1, \cdots, k\}$. Each unit $u \in U$ has number of components and each component belongs to only one unit. We further assume without loss of generality that unit 1's components are $\{1, \cdots, n_1\}$, unit 2's components are $\{n_1 + 1, \cdots, n_2\}$, and so on and finally unit k's components are $\{n_{k-1} + 1 , \cdots, n\}$. For a general notation, unit u's components are $\{n_{u-1} + 1, \cdots, n_u\}$ where $n_0 = 0$ and $n_k = n$.

The matrix of actions A can then be segmented by the participating units, i.e.,

$$A = \begin{pmatrix} A_1 \\ \vdots \\ A_u \\ \vdots \\ A_k \end{pmatrix} = \begin{pmatrix} \bar{a}_1 \\ \vdots \\ \bar{a}_{n_1} \\ \hdashline \vdots \\ \hdashline \bar{a}_{n_{u-1}+1} \\ \vdots \\ \bar{a}_{n_u} \\ \hdashline \vdots \\ \hdashline \bar{a}_{n_{k-1}+1} \\ \vdots \\ \bar{a}_n \end{pmatrix}$$

where A_u, $1 \leq u \leq k$, is given by

$$A_u = \begin{pmatrix} \bar{a}_{n_{u-1}+1} \\ \vdots \\ \bar{a}_{n_u} \end{pmatrix} = \begin{array}{c} A_{u1} \\ \left(\begin{array}{ccc} \boldsymbol{a_{n_{u-1}+1,1}} & \cdots & a_{n_{u-1}+1,N} \\ \vdots & \ddots & \vdots \\ \boldsymbol{a_{n_u,1}} & \cdots & a_{n_u,N} \end{array} \right) \end{array}$$

We will denote by A_{u1} the first column of matrix A_u, i.e., the actions of the components of unit u at time interval $t = 1$ (upcoming time interval). That is,

$$A_{u1} = \begin{pmatrix} a_{n_{u-1}+1,1} \\ \vdots \\ a_{n_u,1} \end{pmatrix}$$

4.2 Market Bids

Every unit $u \in U$ submits a bid to the market

$$\{\langle cost_i, rev_i, intrinsicVal_i, power_i, constr_i \rangle \mid$$

$$n_{u-1} + 1 \leq i \leq n_u\}$$

which gives a tuple $\langle cost_i, rev_i, intrinsicVal_i, power_i, constr_i \rangle$ for every component i of u where:

- $cost_i : \mathrm{dom}(i)^N \to \mathbb{R}^+$ is a function that gives aggregate cost of component i associated with actions $\bar{a}_i$ over the time horizon. For example, a cost of a diesel engine generator consist of the acquisition cost, fuel cost, maintenance cost, etc.

- $rev_i : \mathrm{dom}(i)^N \to \mathbb{R}^+$ is a functions that gives the revenue of operation received (in dollars) of component i associated with actions $\bar{a}_i$ over the time horizon,for example, dispatching the battery to a another unit in returns for compensation.

- $intrinsicVal_i : \mathrm{dom}(i)^N \to \mathbb{R}^+$ is a function that gives the intrinsic value (or utility acquired) of operating component i (in dollars) given the actions $\bar{a}_i$ over the time horizon. In other words, this is the value that unit u is willing to get in lieu of not operating component i.

- $power_i : \mathrm{dom}(i) \to \mathbb{R}$ is a function that gives the power in kW that component i produces (or consumes) given the control actions $\bar{a}_i$ at any time point. A positive value means that the component gives power while a negative value means that the component takes power.

- $constr_i(\bar{a}_i)$ is the operational constraint of component i in terms of control actions $\bar{a}_i$. For example, maximum charge rate (in kW) and maximum discharge rate (in kW) are constraints that affect the power of a battery resource.

We denote the net value of operating component i given control actions $\bar{a}_i$ by the function

$$value_i : \mathrm{dom}(i) \to \mathbb{R}^+$$

which is defined by

$$value_i(\bar{a}_i) \stackrel{\text{def}}{=} intrinsicVal_i(\bar{a}_i) + rev_i(\bar{a}_i) - cost_i(\bar{a}_i).$$

The total value of components given their control action matrix

$$A = \begin{pmatrix} \bar{a}_1 \\ \vdots \\ \bar{a}_z \end{pmatrix}$$

is defined as

$$\mathit{totalValue}(A) \stackrel{\text{def}}{=} \sum_{i=1}^{z} \mathit{value}_i(\bar{a}_i).$$

4.3 Market Resolution and Its Desired Properties

Definition 1. *A market resolution is a set*

$$\{\langle A^*_{u1}, P_u\rangle \mid 1 \le u \le k\}$$

where, for every $1 \le u \le k$:

- *A^*_{u1} is the actions matrix at time interval 1 (upcoming time interval) for all components of unit u .*
- *P_u is the payment amount (in dollars) by unit u. A positive value indicates that unit u makes a payment while a negative value means that the unit receives a payment.*

We propose the following desired properties of a market resolution:

- Feasibility
- Pareto-optimality
- Nash equilibrium
- Equal collaboration profitability

which we describe next.

Intuitively, a market resolution is feasible if the action vector for every unit u at time point 1 can be extended for the entire time horizon without violating unit u's constraints. More Formally,

Definition 2. *We say that a market resolution $\{\langle A_{u1}, P_u\rangle \mid 1 \le u \le k\}$ is feasible if, for every unit u, $1 \le u \le k$, there exists a unit's action matrix*

$$A_u = \begin{pmatrix} \bar{a}_{n_{u-1}+1} \\ \vdots \\ \bar{a}_{n_u} \end{pmatrix} = \begin{pmatrix} a_{n_{u-1}+1,1} & \cdots & a_{n_{u-1}+1,N} \\ \vdots & \ddots & \vdots \\ a_{n_u,1} & \cdots & a_{n_u,N} \end{pmatrix}$$

$$\mathit{where} \begin{pmatrix} a_{n_{u-1}+1,1} \\ \vdots \\ a_{n_u,1} \end{pmatrix} = A^*_{u1}$$

In the definition, note that the actions for time point 1 are exactly those of A^*_{u1} so that each component i of u, $n_{u-1}+1 \le i \le n_u$ satisfies its constraints $\mathit{constr}_i(a_{i,1}, \cdots, a_{i,N})$.

To define the properties of Pareto optimality and Nash equilibrium, we need to define the notions of *selfValue* and *collabValue* for unit u , associated with the market resolution.

Intuitively, a *selfValue* of u is the value u can optimally achieve without collaborating with other units. More formally,

Definition 3. *$\mathit{selfValue}_u$* =

$$\begin{aligned} &\max_{A_u} \quad \mathit{totalValue}(A_u) \\ &\mathit{subject\ to} \\ &(\forall i,\ n_{u-1}+1 \le i \le n_u) \quad \mathit{constr}_i(\bar{a}_i) \quad \wedge \\ &(\forall t,\ 1 \le t \le N) \sum_{i=n_{u-1}+1}^{n_u} \mathit{power}_i(a_{i,t}) = 0 \end{aligned}$$

The optimal actions matrix A^s_u is the actions matrix that gives the maximum self value under the the same constraints. That is,

$$\begin{aligned} A^s_u \in\ &\arg\max_{A_u} \ \mathit{totalValue}(A_u) \\ &\mathit{subject\ to} \\ &(\forall i,\ n_{u-1}+1 \le i \le n_u)\ \mathit{constr}_i(\bar{a}_i) \quad \wedge \\ &(\forall t,\ 1 \le t \le N) \sum_{i=n_{u-1}+1}^{n_u} \mathit{power}_i(a_{i,t}) = 0 \end{aligned}$$

Intuitively, given a market resolution, a collaborative value of u is the value that u can optimally achieve by extending its actions A^*_{u1} from the market resolution. More formally,

Definition 4. *$\mathit{collabValue}_u$* =

$$\begin{aligned} &\max_{A_u} \ \mathit{totalValue}(A_u) \\ &\mathit{subject\ to} \\ &(\forall i,\ n_{u-1}+1 \le i \le n_u)\ \big(\mathit{constr}_i(\bar{a}_i) \wedge (a_{i,1} = a^*_{i,1})\big) \wedge \\ &\sum_{i=n_{u-1}+1}^{n_u} \mathit{power}_i(a_{i,1}) = \sum_{i=n_{u-1}+1}^{n_u} \mathit{power}_i(a^*_{i,1}) \wedge \\ &(\forall t,\ 2 \le t \le N) \sum_{i=n_{u-1}+1}^{n_u} \mathit{power}_i(a_{i,t}) = 0 \end{aligned}$$

where $a^*_{i,1}$ is a component of $A^*_{u_1}$ from market resolution.

The optimal actions matrix A^c_u is the actions matrix that gives the maximum collaborative value for each unit under the the same constraints. That is,

$$
\begin{aligned}
A_u^c \in \arg\max_{A_u} \quad & totalValue(A_u) \\
& \text{subject to} \\
& (\forall i,\ n_{u-1}+1 \le i \le n_u)\ \big(constr_i(\bar{a}_i) \wedge \\
& (a_{i,1} = a^*_{i,1})\big) \wedge \\
& \sum_{i=n_{u-1}+1}^{n_u} power_i(a_{i,1}) = \sum_{i=n_{u-1}+1}^{n_u} power_i(a^*_{i,1}) \wedge \\
& (\forall t,\ 2 \le t \le N) \sum_{i=n_{u-1}+1}^{n_u} power_i(a_{i,t}) = 0
\end{aligned}
$$

Intuitively, a market resolution is *Pareto-optimal* if no other market resolution can increase the value of a unit without decreasing the value of another unit. More specifically,

Definition 5. *Pareto-ptimality: We say that a market resolution* $\{\langle A^*_{u_1}, P_u\rangle \mid 1 \le u \le k\}$ *is Pareto-optimal if there does not exist a market resolution* $\{\langle A'_{u_1}, P'_u\rangle \mid 1 \le u \le k\}$ *such that*

$$(\forall u \in U) \quad (collabValue'_u + P'_u) \ge (collabValue_u + P_u)$$

and

$$(\exists u \in U) \quad (collabValue'_u + P'_u) > (collabValue_u + P_u)$$

no other market resolution can increase the value of a unit without decreasing the value of another unit.

Definition 6. *Nash equilibrium: We say that a market resolution satisfies the Nash equilibrium property if, no unit can get a higher value by quitting the coalition, i.e.,*

$$(collabValue_u - P_u) \ge selfValue_u$$

Definition 7. *Equal collaboration profitability of market resolution (fairness): We say that a market resolution satisfies the equal collaboration profitability property if every unit u has the same profit margin* r_u*, defined as*

$$r_u \stackrel{\text{def}}{=} \frac{(collabValue_u - P_u) - selfValue_u}{selfValue_u}$$

Note that $collabValue_u - P_u$ reflects the total value that unit u receives from the market (*collabValue* minus the payment).

Definition 8. *Market-Resolution Algorithm properties: We say that a market resolution algorithm satisfies the properties of (1) Feasibility, (2) Pareto-optimality, (3) Nash equilibrium, and (4) Equal collaboration profitability, if for every market setting and market bids, it returns a market resolution that satisfies the corresponding properties.*

5 Market Resolution Algorithm

After we have defined the market resolution and its desired properties, we now present how our market resolution algorithm is derived. We first define the global optimization upon which the control actions of the market resolution for the upcoming time interval are based(A^*_{u1}). We then extend these control actions for all the units to find their collaborative value and then compare it to their non-collaborative value and calculate the added benefit of collaboration (Δ). Finally we define how this added benefit is distributed in order to determine the payment (P_u) that each unit has to give or receive , which completes the market resolution.

5.1 Global Optimization

The optimal value that the coalition can achieve which maximizes the welfare is given by the maximization

$$
\begin{aligned}
globalValue = \max_{A} \quad & totalValue(A) \\
& \text{subject to} \\
& (\forall i,\ 1 \le i \le n) constr_i(\bar{a}_i) \quad \wedge \\
& (\forall t,\ 1 \le t \le N) \sum_{i=1}^{n} power_i(a_{i,t}) = 0
\end{aligned}
$$

The control actions matrix that produces the optimal maximum total value for all components is the global control actions matrix A^g, i.e.,

$$
\begin{aligned}
A^g \in \arg\max_{A} \quad & totalValue(A) \\
& \text{subject to} \\
& (\forall i,\ 1 \le i \le n) constr_i(\bar{a}_i) \quad \wedge \\
& (\forall t,\ 1 \le t \le N) \sum_{i=1}^{n} power_i(a_{i,t}) = 0
\end{aligned}
$$

The matrix

$$A^g = \begin{pmatrix} a^g_1 \\ \vdots \\ a^g_n \end{pmatrix} = \begin{pmatrix} a^g_{1,1} & \cdots & a^g_{1,N} \\ \vdots & \ddots & \vdots \\ a^g_{n,1} & \cdots & a^g_{n,N} \end{pmatrix}$$

represents the control actions that all units collectively need to make in order to achieve the optimal value over the entire time horizon. However, since our market executes at every time interval, we are only interested in the upcoming time interval ($t = 1$). The control action matrix by units where A^g_u, $1 \le u \le k$, is given by

$$A^g_u = \begin{pmatrix} \bar{a}^g_{n_{u-1}+1} \\ \vdots \\ \bar{a}^g_{n_u} \end{pmatrix}$$

The market resolution control actions are therefore adopted from the this global optimization, i.e.,

$$A^*_{u1} = A^g_{u1} = \begin{pmatrix} a^g_{n_{u-1}+1,1} \\ \vdots \\ a^g_{n_u,1} \end{pmatrix}$$

5.2 Added Collaboration Benefit (Δ)

After determining the optimal market resolution control actions, the impact on units' values for choosing this resolution must be measured in order to compensate the units appropriately. The additional value the units collectively get by collaborating is the sum of the their collaborative values minus the sum of their non-collaborative values, i.e.,

$$\Delta = \sum_{u=1}^{k} collabValue_u - \sum_{u=1}^{k} selfValue_u$$

We assume that each unit u has a non-negative share of this Δ, i.e.,

$$\Delta = \sum_{u=1}^{k} \Delta_u \qquad , \ (\forall u = 1, \cdots, k) \quad \Delta_u \geq 0$$

Value difference is the value that each unit u gets by participating versus working alone, i.e.,

$$V_u = collabValue_u - selfValue_u$$

P_u is the payment that unit u makes ($P_u < 0$ means that u receives payment)

$$\Delta_u = V_u - P_u$$

Therefore,

$$P_u = V_u - \Delta_u$$

5.3 Added Benefit Distribution

The only remaining part is to find a *fair* methods to calculate Δ_u. We recall that we use the principle of *equal collaboration profitability* which means that each unit gets a portion of the resulting added value of collaboration proportional to its standalone value, i.e.,

$$r = \frac{\Delta_1}{selfValue_1} = \cdots = \frac{\Delta_k}{selfValue_k}$$

where r is the ratio of the *equal collaboration profitability* margin. That is,

$$(\forall u \in U) \qquad \Delta_u = r \cdot selfValue_u$$

Since,

$$\Delta = \sum_{u=1}^{k} \Delta u$$

Therefore,

$$\Delta = \sum_{u=1}^{k} r \cdot selfValue_u$$

Thus,

$$\Delta = r \cdot \sum_{u=1}^{k} selfValue_u$$

Finally,

$$r = \frac{\Delta}{\sum_{u=1}^{k} selfValue_u}$$

The market resolution algorithm is summarized in Algorithm 1.

Algorithm 1 Market Resolution

Input: Market setting, Market bids
Output: Market resolution $\{\langle A^*_{u_1}, P_u\rangle \mid 1 \leq u \leq k\}$

```
Let optCont[u] = ∅, pay[u] = ∅, V[u] = ∅
Let totalSelfValue = ∅, totalCollabValue = ∅
Let Δ = ∅, r = ∅, Δ[u] = ∅
Solve globalValue
for u ← 1 to k do
    optCont[u] ← A^g_{u1}
end for
for u ← 1 to k do
    Solve selfValue_u
    Solve collabValue_u
    V_u ← collabValue_u − selfValue_u
    totalSelfValue ← totalSelfValue + selfValue_u
    totalCollabValue ← totalCollabValue + collabValue_u
end for
Δ ← totalCollabValue − totalSelfValue
r ← Δ/totalSelfValue
for u ← 1 to k do
    Δ_u ← r × selfValue_u
    pay[u] ← V_u − Δ_u
end for
return (optCont, pay)
```

Theorem 1. *The Market Resolution algorithm guarantees the desired market properties which are:*

- *Feasibility*
- *Pareto-optimality*
- *Nash equilibrium*
- *Equal collaboration profitability*

Proof. Feasibility follows directly from A^c_u where actions of the global optimization are extended to satisfy the constraints of individual units. Pareto-optimality follows from the fact of the collaborative maximization of the consortium value ($collabValue_u$). *Nash* equilibrium follows from the fact that $\Delta \geq 0$, and thus $\Delta_u \geq 0$ for every $u \in U$. Equal collaboration profitability of the added value of collaboration follows directly from the way the payment P_u and thus Δ_u is distributed. $\square$

6 Conclusions & Future Work

This paper introduced an extensible decision-guided market-based framework where units that contain power components can collaborate to increase their value and reduce their cost. This market described the market setting, market bids, and market resolution. This framework also described a number of desired properties the market resolution algorithm must satisfy. This paper leveraged our prior work on modular modeling and optimization techniques such as [22, 23, 24, 25, 26, 27]

This up to our knowledge is the first attempt to model a generic electric power collaboration market with multiple players which is extensible. We plan in the future to implement a comprehensive study where we define specific instances of component's controls and constraints and model them using an optimization language such as Optimization Programming Language (OPL). We also plan to investigate whether the market mechanism can be *gamed* by participants, by trying to infer the bid of other participants, e.g., using techniques from [28].

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