Syllabus & Assignments for COMP 502 Spring 2024
Mathematical Foundations of Computer Science

Instructor	Prof. William D. Ellis, wellis1@gmu.edu
Office Hours	By appointment, usually via Blackboard/"Video chat…" Thur 4-5 PM
TA	Mr. Vipin Narahari, vnarahar@gmu.edu
Office Hours	4-5 PM in ENGR 4456 on Friday 1/26/2024 & Thursdays starting 2/1
Web Site	Syllabus/HW updates, practice problems, solutions, notes etc. are delivered via Blackboard: http://mymason.gmu.edu.
Schedule	14 Classes 7:20-10:00 PM Innovation 204 • Wednesdays Jan. 17 through April 24 <u>except</u> Mar. 6, 2024 • The Final Exam will be 7:30-10:15 PM on Wednesday May 1, 2024
Prerequisite	A working knowledge of algebra. Review textbook pages A1-A3.
Course Content	Logic, set theory, sequences, algorithm complexity, probability, recursion, number theory. We will focus on solving problems using fundamental definitions, theorems, and algorithms. We'll follow the textbook in this order: Chs. 5, 4, 11, 9, 6-8, 2-3. Examples include: Fibonacci numbers, RSA cryptography, P vs. NP problem.
Required Hardware	(i) A calculator capable of displaying 10 digits and raising numbers to powers. Homework, Quizzes and Exams, often require such a calculator. (ii) You'll need a webcam for any Office Hour appointment or if and when bad weather pushes a lecture online.
Graded Exams and Quizzes	We will have: (i) 2 Quizzes, (ii) 2 Hour Exams, and (iii) 1 comprehensive Final Exam. The quizzes and exams: • Will be given only once (no makeup Quizzes or Exams), • No partial credit for attempts at proving a false statement. During Quizzes & Exams: • You may <u>not</u> provide, accept, or seek help from anyone else. • Calculations must be based on your calculator and may <u>not</u> be derived from the internet, a computer, or smart watch etc. • You may use typewritten notes and other typewritten papers, including copies of pages from the textbook. • You may use computer-printed copies of handwritten notes. • Do <u>not write</u> on anything except your exam or on any blank pages that are part of your original pre-printed exam. • Do <u>not</u> use or show any other papers with original handwriting. • Do <u>not</u> use or display a notebook that has original handwriting. • Do <u>not</u> provide, receive or exchange with anyone else any materials including papers, erasers, and calculators. • Do <u>not</u> sit near friends. I will move some students randomly. • Do <u>not</u> use or display cell phones, computers, smart watches.
Textbook	Discrete Mathematics with Applications, 5th ed. By Susanna S. Epp, ISBN-10: 1337694193; ISBN-13: 978-1337694193; Cengage MA • Rental of the e-book version, at a $47.46 discounted rate, is automatically included in your GMU tuition bill. If you "opt out" of the e-book rental no later than Feb. 6, 2024, then the $47.46 charge should be refunded to you. • View by clicking on "e-Textbook access" in Blackboard. • See also a 1/16/2024 GMU Bookstore email, subject "First Day"
Final grade = weighted average of letter grades	45% - Final Exam 40% - 2 Hour Exams X 20% each 15% - 2 Quizzes X 5% each + Homework 5% Final Grades will be posted on Patriot Web, <u>not</u> on Blackboard.
Help & Email	Questions? Email me via GMU email. <u>Never</u> email me via Blackboard/email. Instead, use GMU email for all emails related to GMU. We may also use Blackboard, "Video chat with instructor."

Homework — Upload a black/white pdf no more than 30 minutes after class. Late H/W submissions will not be accepted. Credit is given for trying an assigned problem, even if the answer isn't correct. H/W is assigned after each lecture except the last lecture (on Apr 24). Only the highest 12 of the 13 H/W scores are counted.

Honor Code — Honor Code allegations are resolved by the Honor Committee. See https://oai.gmu.edu/resource-center/ for typical HC sanctions. Beware: Honor Code sanctions may be severe! The Honor Code is at https://oai.gmu.edu/full-honor-code-document/. For COMP 502 this semester, submitting homework based on collaboration and/or classroom discussion is okay.

Semester Schedule

Lecture	Date	Event	All event dates are subject to change
(1)	Jan 17, 2024	1st class	
(2)	Jan 24, 2024		
(3)	Jan 31, 2024		
(4)	Feb 07, 2024	Quiz, Lecture	
(5)	Feb 14, 2024		
(6)	Feb 21, 2024		
(7)	Feb 28, 2024		
	Mar 06, 2024	No Class!	Spring Recess
(8)	Mar 13, 2024	Exam, Lecture	
(9)	Mar 20, 2024		
(10)	Mar 27, 2024	Quiz, Lecture	
(11)	Apr 03, 2024		
(12)	Apr 10, 2024		
(13)	Apr 17, 2024		
(14)	Apr 24, 2024	Exam, Lecture	
(15)	May 01, 2024	FINAL EXAM	

Row	§	Homework is from the textbook or as cited below.	Due
(1)	1.2	#7(b),(e),(f); #9(c)-(h) (page 14) <u>Hint</u>: See textbook pages 7-8 and Examples 1.2.1, 1.2.2, 1.2.4, and 1.2.8 on Blackboard.	HW-1 due 1/24/2024
(2)	5.1	7, 16, 32, 57*, 61 (pages 273-274) * #57: Simply calculate the sum for n=5. Don't bother with the part about "changing variable." <u>Hint</u>: On 5.1.57, see Example 5.1.55 on Blackboard.	HW-1 due 1/24/2024
(3)	5.2	#23, 27, 29. (pg 288) <u>Hints</u>: • #23 Compare with Example 5.2.20 (pg 288) • #27, 29: See the textbook examples on pages 281 and 285, or see Blackboard Example 5.2.25(b), or I recommend the word formula in "Notes On Defining and Summing Sequences" on Blackboard.	HW-1 due 1/24/2024
(4)	5.1	True or False? Why? "∀" means "for all." $\sum_{k=1}^{k=n}\left(8k^3+3k^2+k\right)=n\left(n+1\right)^2\left(2n+1\right)\ \forall\, n\in\{1,2,3,4\}$	HW-1 due 1/24/2024
(5)	<u>Hint</u>: Row (4) asks us to verify 4 formulas, one each for n=1,2,3,4: • For n=1, does $8*1^3+3*1+1 = 1*(1+1)^2*(2*1+1)$? • For n=2, does $(8*1^3+3*2+2)+(8*2^3+3*2+2) = 2*(2+1)^2*(2*2+1)=90$, ...?		
(6)	1.2	12 (pg 15) <u>Hint</u>: Compare with Blackboard solution #1.2.11.	HW-2 due 1/31/2024
(7)	5.1	83 (pg 275) <u>Hint</u>: See #5.1.81 on Blackboard.	HW-2 due 1/31/2024
(8)	5.2	Express $S=\sum_{k=29}^{k=123}(1.6)*\left(\frac{25}{24}\right)^{-k}$ as a decimal number accurate to within .01. For example, your answer might look like "S = 52.33." <u>Hints</u>: • You're adding 95 actual numbers. Compute a few of them to judge the sum's approximate size. • Use Theorem 5.2.2 on page 283, or use the word-formula on page 4 of the BlackBoard pdf "Notes On Defining and Summing Sequences." • See the Solution to Sample Quiz-1, #4.	HW-2 due 1/31/2024
(9)	5.6	8, 14 (page 337) <u>Hints</u>: • 5.6.8 is like Example 5.6.6 on Blackboard. • 5.6.14: See hint on Blackboard & Example 5.6.13.	HW-2 due 1/31/2024
(10)	5.7	2(b)&(d), 4, 25 (pages 350-351) <u>Hint</u>: Blackboard has a hint on 5.7.2(d) plus solved examples 5.7.1(c) & 5.7.7.	HW-2 due 1/31/2024
(11)	5.8	12, 14 (page 363) See the HW-2 hints in Row (12) below.	HW-2 due 1/31/2024
(12)	<u>Hints</u>: • For #5.6.14, see the 5.6.13 solution on Blackboard. • #5.8.12 & #5.8.14 are like the Blackboard solutions to #7 and #8 for Sample Quiz 1. Also see "4 Sample Recurrence Relations Solved." • #5.8.12 & #5.8.14 use Theorems 5.8.3 (pg 357) and 5.8.5 (pg 361). • The Blackboard solution to Sample Quiz 1 #7 shows how to factor the Characteristic Equation using standard methods.		

Row	§	Homework is from the textbook or as cited below.	Due
(13)	1.3	#15(c),(d),&(e) (pg 23) Hint: We already discussed 1.3.15 in class. Also, see Example 1.3.13 on Blackboard.	HW-3 due 2/7/2024
(14)	4.1	4, 9, 13(b) (pages 171-172) Hint: Blackboard #4.1.13(b) is similar to #4.1.14.	HW-3 due 2/7/2024
(15)	4.2	2, 9, 13, 19, 27 (page 181-182). Hints: • For 4.2.9: (1) Call the given integer n. (2) Use the hypothesis on n (i.e., the information given about n) to write an equation: (n-1) = ... (3) Now factor (n-1). (4) Explain why each factor is > 1, thereby showing (n-1) cannot be prime (like in 4.2.14). • For 4.2.19: (i) Identify the error, then state also whether the "Theorem" is TRUE or FALSE, then explain why. (ii) Find the error by comparing the given "proof" with the Blackboard pdf "Bogus proof that 8=10." • For 4.2.13: See the 4.2.14 solution on Blackboard	HW-3 due 2/7/2024
(16)	4.3	7 (pg. 187) Hint: Mimic 4.3.6. We solved this in class and it's solved on Blackboard.	HW-3 due 2/7/2024
(17)	4.1 4.2	Hint: For the section 4.1-4.2 problems, use the even-odd definitions on page 162, NOT the familiar even/odd properties shown in § 4.3 (pages 186-187). Those pg-186-187 properties are derived from the page-162 definitions too!	
(18)	4.3	28 (page 188)	HW-4 due 2/14/2024
(19)	4.4	28, 41 (pages 198-199)	HW-4 due 2/14/2024
(20)	4.5	6, 21, 39 (pages 209-210) Hint: • #39. Break into 6 cases. (Examples 4.5.17 & 37 break into 2 cases. Blackboard has another Hint for #39. • #21 is like #4.5.25 on Blackboard.	HW-4 due 2/14/2024
(21)	4.10	16, 23(b). Hint: For 23(b): See the Hint on Blackboard. Syntax isn't important. In plain English, describe in separate bullets this algorithm's: (1) input, (2) action (what it does with the input), and (3) output.	HW-4 due 2/14/2024
(22)	4.10	Calculate GCD(98741, 247021)	HW-4 due 2/14/2024
(23)	4.10 & 5.8	Write the Fibonacci no. F_{400} in scientific notation, e.g. $F_{30} \approx 1.35*10^{6}$. Use Epp's definition $F_0=1$, $F_1=1$, ... on page 333; or use the formula in # 5.6.33 on page 339, or Solution to Sample Exam 1 #2.	HW-4 due 2/14/2024

Row	§	Homework is from the textbook or as cited below.	Due
(24)	6.3	# Suppose the set S has size $\lvert S\rvert=4$, T is the power set T=P(S) and U is the power set U=P(T). What is the size of U? Your answer should be in a format like, "The size $\lvert U\rvert$=18,419." Hint: See page 410 or the solution to Sample Exam 1 #6 on Blackboard.	HW-5 due 2/21/2024
(25)	6.3	# S = {1, 2, 3, ..., 10}. A subset $X \subseteq S$ contains either 9 or 10 but not both together in X. Exactly how many such subsets X are possible? Hint: Draw a picture (Venn Diagram) for the power set P(S) with 3 bubbles, one each for the subsets X that contain: (i) 9, (ii) 10, and (iii) both.	HW-5 due 2/21/2024
(26)	4.10	# Observe: $247{,}710^2 - 38{,}573^2$ $= 61{,}360{,}244{,}100 - 1{,}487{,}876{,}329$ $= 59{,}872{,}367{,}771 = 260{,}867*229{,}513$. Now factor 260,867 in a non-trivial way. Hints: Mimic the Blackboard factorization examples. "Excel: Euclidean Algorithm" (under Blackboard Week #4) can speed up your calculations.	HW-5 due 2/21/2024
(27)	11.2	# Read Blackboard "Synopsis Of Big-O Notation and Algorithm Complexity" (a 1-page pdf + a table) #27, #33 (page 785), except • For both problems, only determine the complexity level like in Examples 11.2.26 & 11.2.32 • For #33, use the formula in #5.2.11 on pg 287	HW-5 due 2/21/2024
(28)	9.1	#4; #8; #14(b)-(c); #Redo 14(b)-(c) using: 30% for Mr. A; 60% for Mr. B; and 40% for Mr. C. Hints: Mimic Blackboard Examples 9.1.3, 7, 10, 12.	HW-5 due 2/21/2024
(29)	9.2	#7, #17(a)-(d) Hints: • #7: See BB Example 9.2.6, or the 9.2.6 Alternate Solution. • #17 is like BB Example 9.2.12, but #17(d) is tricky! Try choosing the rightmost digit first (5 choices); then the leftmost (8 choices)! (Why start at the right instead of the left?)	HW-5 due 2/21/2024
(30)	4.10	# Suppose, $2^{280} < a < 2^{281}$; $3^{110} < b < 3^{111}$; a,b are integers; and we calculate GCD(a,b) with the Euclidean Algorithm [EA]. According to Lamé's Theorem (page 253), exactly how many EA divisions (i.e. steps) will be needed in the worst case? Hint: See solution (1) in the Blackboard example, "Sample Comparison Worst-Case Steps, Lame's Theorem vs. Fibonacci #."	
(31)	9.2	#33, #39	
(32)	9.3	#32 Hint: See Blackboard "Example: Birthday-Collision Probabilities (based on 366 days)."	

Row	§	Homework is from the textbook or as cited below.	Due
(33)	9.5	7(a)-(b), 10, 12, 16, 20 Hints: • 9.5.7(a)-(b): We did a similar problem, 9.5.6 in class. 9.5.6 is also solved in the textbook. • 9.5.12: Count separately the subsets where: (1) both elements are even, and (2) both are odd. • 9.5.16: 9.5.14 on Blackboard similarly adds and subtracts C(n,r) values. • 9.5.20: See Example 9.5.19 on Blackboard.	
(34)	9.6	#4 Hints: • C(r+n-1,r) = C(r+n-1,n-1) is the number of ways for selecting r objects (repetitions allowed) from among n varieties. The text differentiates r and n not so well - see the theorem on page 636! • See the Blackboard solution to 9.6.3.	
(35)	9.6	#13 Hint: See the solution on Blackboard to 9.6.12.	
(36)	9.6	5 people are chosen at random. Calculate P = the probability that one (1) has birthday February 29, three (3) have birthday April 1, and one (1) has birthday October 31. Assume 366 days in every year, and individual probabilities are equal (1/366) for being born on every date. Hint: 366^5 is the size of the sample space, the Cartesian Product S x S x S x S x S, where S = {1,2,...,366}. Now see page 629.	
(37)	9.7	#24, 34 Hint: Mimic Blackboard Examples 9.7.23, 9.7.33	
(38)	9.8	#18, #20 Hints: See Blackboard solutions to 9.8.17, 9.8.19, and #3 Practice Problem Quiz 2 (Gambler's Exp. Val.	
(39)	9.9	#2, #12. Hints: • For #2, see the Blackboard solution to 9.9.1 • For #12, see the Blackboard solution to 9.9.11 and/or the "viral infections" example.	
(40)	9.9	#5 Practice Problem for Quiz 2 (Conditional probability & voter genders). Hint: See Blackboard "Yellow Birds" example.	
(41)	9.9	An unfair coin is flipped **8** times. The probability of landing Heads is **75%** on each flip. Question: What is the probability of landing exactly **3** Heads? Hint: A similar problem is solved on pages 671-672, Example 9.9.9(a), except there the unfair coin is flipped 10 times, P(heads)=80% on each flip, and it asks about having exactly 8 successes.	
(42)	9.8	• #4 Practice Problem Quiz 2 (Expected Value-coin). • Read Blackboard Week-8/Class Notes: Binomial Probabilities and ..." and/or Example 9.9.9 pg 672.	

Row	§	Homework is from the textbook or as cited below.	Due
(43)	6.1	Of a population of students taking 1-3 classes each, exactly: 19 are taking English, 20 are taking Comp Sci, 17 are taking Math, 2 are taking only Math, 8 are taking only English, 5 are taking all 3 subjects, and 7 are taking only Computer Science. How many are taking exactly 2 subjects?	
(44)	6.1	#7b; #10(f)-(h); #12(a),(b),(g),(h),(j) (pgs 388-9) Hints: • #7,#10: See 6.1.4, 6.1.10(a)-(e) on Blackboard. • #12: Using Interval Notation (page 382) would vastly simplify your writing. • #12(g): It's alright to use #12(a) and De Morgan laws in the next section § 6.2 (pg 395). This problem makes us appreciate the De Morgan laws.	
(45)	6.1	#7 on Sample Quiz #2	
(46)	6.2	#13. Hint: See Example 6.2.9 on Blackboard	
(47)	6.3	#7 Hint: Numbered Venn-Diagram regions are good for verifying or finding a counterexample to a "∀ sets" identity. See Examples 6.2.9(i) & 6.3.4-5.	
(48)	6.3	Prove or disprove each of these 2 Claims: **(i)** ∃ sets A, B & C such that (A-B)-C=(A-C)-(B-C), **(ii)** ∀ sets A, B & C, (A-B)-C = (A-C)-(B-C). A proof may use any method, including I-III in Example 6.2.9. Any Venn-Diagram arguments must use numbered regions instead of shaded regions. Hint: See 6.3.13 Example on Blackboard.	
(49)	7.1	• #2, #5; #12 (page 436) Hints: See the solution to 7.1.1 on page A-72, and 7.1.11 on Blackboard; also 1.3.20 on Blackboard.	
(50)	7.1	#51(d),(e), and (f) (pgs 436-439) Note: The hint on Blackboard has examples like we saw in class. #51 will be used in RSA encryption.	
(51)	7.2	#13, #17 Hint: Mimic Blackboard Examples #16 and #18. • The "1-1" definition is on page 440, and • The "onto" definition is on page 446. Both concepts are illustrated in Example 7.2.8.	
(52)	7.2	#9 on Sample Quiz #2 An "onto" function is defined on page 446.	
(53)	7.3	#4	
(54)	7.3	#14 - There's a hint on Blackboard. #20. Hint Try drawing a picture; see Example 7.3.19	
(55)	1.3	• #4 (pg 22) Hint: See Example 1.3.3 on Blackboard. Also see the definition of "relation from A to B" and the Example on pg 16. Every one of the arrow diagrams in 1.3.15 (HW-3) represents a relation.	

Row	§	Homework is from the textbook or as cited below.	Due
(56)	8.1	#3(c)-(d). (page 493) Hint: See 8.1.1, solved on Blackboard.	
(57)	8.2	#11 (page 503). Hint: See the solution to 8.2.10 on Blackboard, or "Practice problems: Properties of Equivalence Relations"	
(58)	8.3	#9 [Call 0 = the sum of the elements in φ.]; #15(b),(c),(d) (page 521) Hints: • #9 See Blackboard Examples 8.3.8, 8.3.10, 8.3.12 • #15: Use the modular-equivalence definition on page 518; or Blackboard "Two Reasons Why 76^googol has Remainder 1 When Divided by 77," paragraph 2.	
(59)	8.4	#2, #4, #8 (page 544). Hints: • 8.4.4 is like Example 8.4.3. • 8.4.8 is like Example 8.4.7.	
(60)	8.4	# Calculate 2^{373} (mod 367). Hint: Blackboard "Example: Powers of 3 (mod 257)" or textbook page 531	
(61)	8.4	#18. Hints: For successive squaring examples, see Blackboard "Example: Powers of 3 (mod 257)" or Example, or Example 8.4.17, or the examples on textbook page 531.	
(62)	8.4	#9 Practice Exam-2; remainders (mod 9) & (mod 11). Hint: See Blackboard "Examples 8.4.12 & 8.4.13: Reducing Integers (mod 9) and (mod 11)."	
(63)	8.4	#20 Hint: See Example 8.4.21 on Blackboard. Here, we convert WELCOME into a string of integers like in H/W 8.4.2. Next, reduce each integer x -> e(x) = x^3 (mod 55), e.g., L -> 12 -> 12^3 ≡ 23 (mod 55). This problem mimics Example 8.4.9 on page 537.	
(64)	8.4	#37 See Examples 8.4.21, 8.4.23, 8.4.38, and "RSA Examples: Sample Exam-2 #10, #11 Solutions." Only here we convert COME into a string of integers like in H/W 8.4.2. Next, reduce each x -> e(x) = x^43 (mod 713), e.g., C -> 3 -> 3^43 ≡ 675 (mod 713).	
(65)	8.4	Solve for x: 1014*x ≡ 7 (mod 4,157), 0 ≤ x ≤ 4,156. Hint: See either of these 3 Blackboard Examples: (1), 8.4.27, or (2) Solve 122x = 9 (mod 7919), or (3) Solving 136y = 14 (mod 7919).	
(66)	8.4	#38, 40 Hint: These problems are like Line (65), only now we're solving 43*x≡1 (mod 660). Since the public RSA modulus is 713, if a hacker could calculate φ(731)= 660, it'd be easy to discover the decryption exponent by solving 43*x≡1 (mod 660).	
(67)	8.4	Find the RSA decryption exponent d when: p=13, q=17, n=221, and e=37 is the encryption exponent. Hint: See Blackboard "Example: Creating an RSA Encryption-Decryption Pair when n=821*823."	

Row	§	Homework is from the textbook or as cited below.	Due
(68)	8.4	Solve for x: $x^2 \equiv 4$ (mod 675,683), all 4 solutions must be between 0 & 675,682. Use 675,683 = 821*823, the product of 2 primes. Hint: Solve 821*a+823*b = 1. Then the four sqrt(1) are {+/-1, +/-(821*a-823*b)} (mod 675,683). Now multiply each of the four sqrt(1) by 2, to get four sqrt(4). See Blackboard/Week #11: "Two Examples: Finding four Square Roots (mod pq)."	
(69)	2.1	#15, #37, #43 (pgs 52-53) Hints: •#37 is like #2.1.33 on Blackboard. •#43 is like #2.1.41 on Blackboard.	
(70)	2.2	#4, #15, #27 (pgs 63-64). Hint • 2.2.4: See Black-board "Solution to SE-2 #7, rewriting a ...."	
(71)	2.2	Do "HW Assignment: A small satisfiability ("SAT") problem" on Blackboard (Week #12)	
(72)	2.3	#9 (pg 77) Hints: See Blackboard Week #12: • "Example: Test a syllogism's validity," or • "Example: Sample Exam-2 #4 Solution," or • Example 2.3.11. Epp's shortcut vs. the common-sense method for determining validity are compared in Table 5 of "Truth Tables, Arguments Forms & Syllogisms." But, if at least one premise is FALSE every row of its truth table, would an argument form be valid?	
(73)	4.5	Suppose x is an integer. Consider the statement *s* := "(x^2-x) is exactly divisible by 3." Choose and answer exactly one of (A), or (B), or (C): (A) Prove s is TRUE; or (B) Prove s is FALSE; or (C) Explain why (A) and (B) are both impossible.	
(74)	2.2	Solve "HW Problem on Informal English" posted on Blackboard	
(75)	3.1	#12, #18(c)-(d); #28(a)&(c); #32(b),(d) pgs 119-121 Hints See Blackboard Example 3.1.18 (a),(b),&(e). • 3.1.28: To void remembering order of operations, put a parenthesis before "Real" and after "Neg(x)."	

Row	§	Homework is from the textbook or as cited below.	Due
(76)	3.2	#10, #25(b)-(c), #38 (pages 130-131). In these problems: • ∀ and ∃ are the only quantifiers that may be used here. Do not put any slashes through a quantifier, e.g. do not us a ∄. • No negation symbol (¬) may appear outside a quantifier or an expression involving logical connectives, e.g. instead of "¬(∀x.(P(x)->Q(x))))," write "∃x.(P(x)∧¬Q(x))." Hints: • On #10, see Examples 3.2.9 and/or 3.2.17. • On #38: *Discrete Mathematics* refers to the phrase "Discrete Mathematics," not to the entire subject of Discrete Mathematics.	
(77)	3.3	#41(c),(d),(g),(h) (page 145) Hints: See the Order of Quantifiers example we discussed in class, L(x,y) := "x loves y" (pg 138).	
(78)	3.3	Let s := (∀x.(P(x)∧∃y∃z.Q(x,y,z))) -> (∃x∃y.R(x,y)). Negate s and simplify ¬s so: • No negation symbol (¬) appears outside a quantifier or an expression involving logical connectives(like ∧, ∨, or ->. • Use only the ∀ and ∃ quantifiers. Do not put any slashes through a quantifier, e.g. do not us a ∄. Hint: See "Example: Negating a Multiply-quantified statement" on Blackboard.	