| ctor: | Prof. William D. Ellis E-mail: wellisl@gmu.edu |
| :---: | :---: |
| Office Hours: | By appt. (usually Wednesdays 5-6 PM) 4456 Engineering Bldg. |
| Blackboard/ | Syllabus/HW updates, sample problems \& solutions, lecture notes |
| Web Site: | etc. are posted weekly after class at http://mymason.gmu.edu. |
| Schedule: | 14 Classes 7:20-10:00 PM <br> Arts \& Design Bldg. Room 2026 <br> - Wednesdays except no class on March 13 during Spring Vacation <br> - The Final Exam is Wednesday May 8, 2019 from 7:30-10:15 PM |
| Prerequisite: | "Completion of 6 hours of undergraduate mathematics." As a practical matter, you need a working knowledge of algebra, including the laws of exponents. Several free tutorials may be found on the Internet. Also see textbook Appendix pages A1-A3. |
| Topics: | We will follow the textbook in this order: Chapters 5, 4, 2, 3, $6,7,8,10$, and 9. We will focus on problem solving, and we will use fundamental definitions, theorems, and algorithms. As examples, we'll learn about the $P$ versus NP problem, RSA publickey cryptography, Benford's Law, and the Bitcoin Blockchain. |
| Calculator: | You will need a calculator that can display 10 digits and raise numbers to powers. During an exam or quiz: Do not (1) use a computer or cell phone, or (2) share anything with others. |
| Textbook: | Discrete Mathematics with Applications, 4th ed. (8/4/2010) By Susanna S. Epp, ISBN-10: 0495391328; ISBN-13: 978-0495391326. No e-books may used during any quiz or exam. |
| Exams and | We will have: (i) 2 Quizzes, (ii) 2 Hour Exams, and (iii) a |
| Quizzes: | comprehensive Final Exam (Wednesday May 8, 2019). Exams and |
|  | Quizzes will be given only once - no makeup exams. Use all available classroom space, avoid sitting close to anyone else, and do not sit next to a friend. No partial credit will be given |
|  | for a purported proof to a false statement. During exams and quizzes do not use or display cellphones, computers, or smart watches. Do not share calculators or anything else. Exams and quizzes will be open-book and open-notes. |
| Grades: | 1 Final Exam: 45\% of final grade. <br> 2 Hour Exams: 40\% of the final grade ( $20 \%$ each) |
| Help: | Questions? Send me an e-mail! Use the ^ symbol for exponents, * for multiplication. You may also e-mail a pdf or scanned image. |
| Homework: | Homework assignments will be on the weekly Syllabus updates. See |
|  | However, 13 HW assignments will be turned in, but only the 12 with the highest scores will be counted toward your grade. Submit on paper, please. If you cannot attend class, scan as a black/white pdf and e-mail. NO grey-scale scans, please! |
| Honor Code: | Honor Code violations are reported to the Honor Committee. See http://cs.gmu.edu/wiki/pmwiki.php/HonorCode/CSHonorCodePolicies |
|  | Collaborating on homework or submitting solutions based on classroom discussion is okay but only for INFS501 in Spring 2019. |
| E-mail: | Use only GMU email for all emails with me. |
| Syllabus and H | assignments posted after each class. Rev 12/30/2018 (4:55 PM) |

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Semester Schedule: Dates \& data for Quizzes 1-2 and Exams 1-2 may change.

| Class | Date | Event | Details - some are tentative |
| :---: | :---: | :---: | :---: |
| $(1)$ | Jan 23, 2019 | 1st Class |  |
| $(2)$ | Jan 30, 2019 |  |  |
| $(3)$ | Feb 6, 2019 |  |  |
| $(4)$ | Feb 13, 2019 |  |  |
| $(5)$ | Feb 20, 2019 | Quiz 1 |  |
| $(6)$ | Feb 27, 2019 |  |  |
| $(7)$ | Mar 6, 2019 |  |  |
| $(8)$ | Mar 13, 2019 20, 2019 | no class! |  |
| (9) Lecture | Mar 27, 2019 |  |  |
| $(10)$ | Apr 3, 2019 |  |  |
| $(11)$ | Apr 10, 2019 | Quiz 2 |  |
| $(12)$ | Apr 17, 2019 |  |  |
| $(13)$ | Apr 24, 2019 |  |  |
| $(14)$ | May 1, 2019 | Hour Exam 2 <br> \& Lecture |  |
| $(15)$ | May 8, 2019 | FINAL EXAM |  |


| Row | § | Homework from the textbook or written out below. | Due |
| :---: | :---: | :---: | :---: |
| (1) | 5.1 | 7, 13, 16, 32, 57*, 61, 83 <br> * For 5.1.57, only calculate the sum for $n=5$. Don't bother changing variable like the problem asks. | HW-1 due $1 / 30 / 2019$ |
| (2) | 5.2 | 23, 27, 29. Hint: Try using Example 5.2.2 on page 251 \& Example 5.2.4 on page 255. | HW-1 due 1/30/2019 |
| (3) | 5.2 | False or  <br> True? Why?  <br> $\forall$ Means  <br> $\forall f o r ~ a l l " ~$ $\sum_{k=1}^{n}\left(8 k^{3}+3 k^{2}+k\right)=n(n+1)^{2}(2 n+1) \forall n \in Z^{+}$ | HW-1 due $1 / 30 / 2019$ |
| (4) | 5.2 | Express $S=\sum_{k=29}^{k=123}\left(\frac{25}{24}\right)^{-k}$ as a decimal number with at least two decimal digits of accuracy. For example, your answer might look like "S = 52.33." <br> Hints: • You're adding 53 numbers. Compute a few of them to judge what the sum should look like. <br> - Use Theorem 5.2 .3 on page 253 , or use the wordformula in the "Geometric-Series Summation Formula Generalized \& Simplified" pdf on BlackBoard. <br> - A solved example is \#3 in Sample Quiz 1 on Blackboard. |  |
| (5) | 5.2 | 11, 12 |  |
| (6) | 5.6 | 8, 14, 33 |  |
| (7) | 5.7 | $2(\mathrm{~b}) \&(\mathrm{~d}), 4,25$ |  |
| (8) | 5.8 | 12, 14 |  |
| (9) |  | Hints: <br> - \#5.8.12 \& \#5.8.14 are like the problems \#6 \& \#7 on Sample Quiz 1. <br> - \#5.8.12 \& \#5.8.14 use Theorems 5.8.3 (pg 321) and 5.8.5 (pg 325). <br> - How to factor any Characteristic Equation is explained in the solution to \#6 on Sample Quiz 1. |  |
| (10) | 4.1 | 3, 5, 8. Follow § 4.1 and do not rely on the well-known even/odd properties on page 167 in § 4.2. (The $\$ 4.2$ properties are based on $\$ 4.1$ too!) |  |
| (11) | 4.1 | 12, 27, 36, 50 |  |
| (12) | 4.2 | 2, 7, 28 <br> Hint on \#28: Write $r$ is the format of a rational number, following the definition of "rational." Then use the algebra of fractions to show $3 r^{\wedge} 2-2 r+4$ can be expressed in the same format. |  |
| (13) | 4.3 | 5, 21, 41 |  |

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| Row | § | Homework from the textbook or written out below. | Due |
| :---: | :---: | :---: | :---: |
| (14) | 4.4 | 6, 17, 21, 35, 42 <br> Hints: \#17 is like \#4.4.19 on Blackboard. <br> \#21 is like \#4.4.25 on Blackboard. <br> \#35 is like \#4.4.43 on Blackboard. <br> \#42 is like \#4.4.30 on Blackboard |  |
| (15) | 4.8 | 12, 16; 20 (b) [Don't worry much about syntax. To describe an algorithm, we must describe: (i) its input, (ii) what it says to do, and (iii) its output.] |  |
| (16) | 4.8 | Observe: $\begin{aligned} 247, & 710^{2}-38,573^{2} \\ & =61,360,244,100-1,487,876,329 \\ & =59,872,367,771=260,867 * 229,513 . \end{aligned}$ <br> Now factor 260,867 in a non-trivial way. <br> Hint: See the Hint on Blackboard. |  |
| (17) | 4.8 | Find GCD (98741, 247021) |  |
| (18) | $\begin{gathered} 4.8 \\ 5.8 \end{gathered}$ | Write the Fibonacci no. $\mathrm{F}_{400}$ in scientific notation, e.g. $\mathrm{F}_{30} \approx 1.35 * 10^{6}$. Note: Be careful using and formulas on the Internet. Epp defines the Fibonacci sequence starting with $\mathrm{F}_{0}=1, \mathrm{~F}_{1}=1$ while some other authors (like Wikipedia) start with $\mathrm{F}_{1}=1, \mathrm{~F}_{2}=1$. |  |
| (19) | 2.1 | 15, 37, 43 (page 38) <br> Hints: \#43 is like \#2.1.41 on Blackboard. <br> \#37 is like \#2.1.33 on Blackboard. |  |
| (20) | 2.2 | 4, 15, 27. \#4 is like \#19 on the Sample Exam. |  |
| (21) | 2.3 | 10, 11 (page 62) Hints: <br> - These problems are like \#7 solved in the Sample Exam on Blackboard. <br> - Epp's shortcut method and the common-sense method for determining validity are compared in Table 5 of "Truth Tables, Arguments Forms \& Syllogisms." |  |
| (22) | 4.4 | Suppose we are given an integer $x$. Now call the statement $s="\left(x^{2}-x\right)$ is exactly divisible by 3." Choose one of the answers $A, B$, or $C$ and either: <br> (A) Prove $s$ is TRUE; <br> (B) Prove $s$ is FALSE; or (C) Explain why <br> (A) and <br> (B) are impossible. |  |
| (23) | 2.2 | Posted on Blackboard/Content/Week 6 are two (2) of the ten (10) HW-6 Problems due 3/6/2019 |  |
| (24) | 3.1 | 12, $18(\mathrm{c})-(\mathrm{d}), 28(\mathrm{a}) \&(\mathrm{c}), 32(\mathrm{~b}) \&(\mathrm{~d}) \quad(\mathrm{pgs} \mathrm{106-108)}$ |  |
| (25) | 3.2 | 10, $25(\mathrm{~b})-(\mathrm{c}), 38$ (pages 116-117). <br> Note: In \#38, "Discrete Mathematics" refers to the phrase "Discrete Mathematics," not to the subject of Discrete Mathematics. |  |
| (26) | 3.3 | \#41 (c), (d), (f), (g), (h) (page 130) |  |
| (27) | 3.3 | Sample Quiz-2, \#1 \& \#2 |  |

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| Row | § | Homework from the textbook or written out below. | Due |
| :---: | :---: | :---: | :---: |
| (28) | 1.2 | $\begin{aligned} & \# 7(\mathrm{~b}),(\mathrm{e}) \&(\mathrm{f}) ; \# 9(\mathrm{c})-(\mathrm{j}) ; \# 12 \\ & \text { (Section } 1.2 \text { fits with Ch. } 6 \text { on Set Theory.) } \end{aligned}$ |  |
| (29) | 6.1 | \#7b; \#12(a), (b), (g)\&(j); \#13; \#18, \#33 <br> Hints: <br> \#7 See the Hint on Blackboard. \#7 is like 6.1.4. <br> \#12: Writing $[-3,2)$ for "-3 $<=x<2 "$ etc. is OK. \#12 makes us want the Set Identities on pg 355. <br> \#13 See the Hint on Blackboard. <br> \#33 Predict the size of each power set using the theorem on page 369: size $\|S\|=n=>\|P(S)\|=2^{\wedge} n$. |  |
| (30) | 6.1 | Of a population of students taking 1-3 classes each, exactly: 19 are taking English, 20 are taking Comp Sci, 17 are taking Math, 2 are taking only Math, 8 are taking only English, 5 are taking all 3 subjects, and 7 are taking only Computer Science. How many are taking exactly 2 subjects? |  |
| (31) | 6.2 | \#10, \#14, \#32 <br> Hints: Hints for \#14 and \#32 are on Blackboard |  |
| (32) | 6.3 | \#2, \#4, \#7 <br> Hints: Hints for \#2, \#4 are on Blackboard. <br> - Venn-Diagram shading is not acceptable. Shading alone is usually confusing \& unconvincing. <br> - Numbered Venn-Diagram regions may be used to verify or find a counterexample to a " $\forall$ sets" identity. <br> - "Is-an-element" proofs also work for verifying " $\forall$ sets" identities but they' re often complicated. |  |
| (33) | 6.3 | Prove or disprove each of these 2 Claims: <br> (i) $\exists$ sets $A, B \& C$ such that $(A-B)-C=(A-C)-(B-C)$, <br> (ii) $\forall$ sets $A, B \& C,(A-B)-C=(A-C)-(B-C)$. |  |
| (34) | 1.3 | $\begin{aligned} & \# 15(c),(d), \&(e) ; \# 17 . \text { (pg 22) } \\ & \text { These tiny problems fit with Ch. } 7 \text { on Functions. } \end{aligned}$ |  |
| (35) | 7.1 | \#2; \#5; \#51(d), (e), \& (f) (pg 393) <br> Note: \#51 Will be used in RSA encryption. |  |
| (36) | 7.2 | See the "H/W-9 Hash Function Problem" on Blackboard |  |
| (37) | 7.2 | 8, 13 (b) , 17 |  |
| (38) | 7.3 | 2, 4, 11, 17 |  |
| (39) | 8.1 | \#3 (c) \& (d) . Hint: See 8.1.1, solved on Blackboard. |  |
| (40) | 8.3 | ```#9 [0= the sum of the elements in the empty set \varphi.] #15 (b), (c),(d) Hints: #9: Like #8, 10, & 12, solved on Blackboard. #15: Use the definition on page 473.``` |  |
| (41) | 8.4 | 2, 4, 8, 17, 18 |  |

Syllabus and HW assignments posted after each class. Rev 12/30/2018 (4:55 PM)

| Row | § | Homework from the textbook or written out below. | Due |
| :---: | :---: | :---: | :---: |
| (42) | 8.4 | Calculate $2^{373}(\bmod 367)$. [Hint: If it matters, 2, 367, and 373 are all prime numbers.] |  |
| (43) | 8.4 | 12b, 13b [Hint: If we call the hundred's digit "h," the tens digit "t," and the unit's digit "u," then the 3 -digit base-10 number htu $=h * 10^{\wedge} 2+t * 10+u$. For 12b, reduce the $10^{\prime \prime} \mathrm{s}(\bmod 9)$. For 13b, reduce the 10's (mod 11). The same approach works no matter how many digits a positive integer has.] |  |
| (44) | 8.4 | Solve for $\mathrm{x}: 1014 * \mathrm{x} \equiv 7(\bmod 4,157), 0 \leq \mathrm{x} \leq 4,156$. |  |
| (45) | 8.4 | $\# 20,21,23,37,38,40$. Hints: <br> \#20-21 use Example 8.4.9: encryption $e=3$ (mod 55). <br> For example, $H=8->8^{\wedge} 3=17(\bmod 55)$. <br> \#23 uses Example 8.4.10: decryption $d=27(\bmod 55)$. <br> For example $17->17 \wedge 27=8(\bmod 55)$. <br> Examples 8.4.9-8.4.10 reverse each other, e.g. <br> (mod 55) $\mathrm{H}=8->17$ (encrypt) $->8=\mathrm{H}$ (decrypt) <br> The pair $(e, d)=(3,27)$ reverse each other because $3 * 27=1(\bmod 40)$ and $40=(5-1)(11-1)=40$ is the <br> Little Fermat exponent (mod 55). <br> \#40 Modulus $=713=23 * 31$ \& encryption $e=43$ are given. From \#38, $43 * 307=1(\bmod (23-1)(31-1))$, so use decryption $d=307$. |  |
| (46) | 8.4 | Under RSA: $p=13, q=17, n=221, \& e=37$ is the encryption exponent. Find $d=$ decryption exponent. See Blackboard, "Example: Creating an RSA Encryption-Decryption Pair." We also solved Sample Exam 2 \#10. |  |
| (47) | 8.4 | Solve for $x: x^{2} \equiv 4(\bmod 675,683)$. Give all 4 solutions. All 4 answers should be between 0 \& 675,682 . Use $675,683=821 * 823$, the product of 2 prime numbers. Hint: See "Square roots (mod pq) two examples.pdf," on BlackBoard. <br> This shows multiple square roots exist under a composite modulus. Multiple square roots allow factoring the RSA modulus as in Row (16) above. RSA is attacked by finding multiple square roots mod the public modulus $n$, to factor $n=p q-t h a t ' s$ the hard part. Then an RSA-cracker only needs to solve $d=e^{-1}(\bmod (p-1)(q-1))$. |  |
| (48) |  | What integer x satisfies: (a) $1 \leq \mathrm{x} \leq 2,622,187$; <br> (b) $x=510(\bmod 661)$; and (c) $x=479(\bmod 3967)$ ? Here, 661*3967 = 2,622,187. <br> Hint: See Blackboard, either: (1) The solution to SE2 \#22.5, (2) "Example: Simultaneous Equations and the Chinese Remainder Theorem," or (3) Section 3.3 on page 8 of the lecture notes, "Summary: Little Fermat, RSA, \& Chinese Remainder Theorem." |  |
| (49) | 10.1 | 4, 19, 20, 29, 34 (pages 639-640) |  |
| (50) | 10.2 | $8(\mathrm{~b}),(\mathrm{c}) \&(\mathrm{~d})$; 9; 10 (pages 657-658) |  |

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| Row | § | Homework from the textbook or written out below. | Due |
| :---: | :---: | :---: | :---: |
| (51) | 10.4 | \#4, \#11, \#13, \#15. <br> On 4, 11, \& 13, explain why the given pair of graphs cannot be isomorphic. <br> Hints: \#13: Look for circuits of length 5. <br> \#15: There are 11 non-isomorphic simple graphs with 4 vertices. |  |
| (52) | 9.1 | 10, 12 (b) (ii)-(iii), 14 (b)-(c), 20 |  |
| (53) | 9.2 | 7, $17(\mathrm{a}),(\mathrm{b}) \&(\mathrm{~d}), 33$ |  |
| (54) | 9.5 | 7(a)-(b), 12 |  |
| (55) | 9.8 | HW-14 will be done in class on 5/1/2019. HW-14 consists of probability problems related to the Blockchain, |  |
|  |  |  |  |
|  |  |  |  |

